

# **Corex** : Pushing the Boundaries of Complex Reasoning through Multi-Model Collaboration

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1st Conference on Language Modeling (COLM) [ArXiv]

# **Background:** Reasoning



## **Chain-of-Thought Reasoning (CoT)**

- "Let's think step by step"
- breaking down problems.
- Solutions are found step-by-step.



Indirect target:  $P(y|x) = P(y|z_n) \cdots P(z_1|x)$ 

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# **Background:** Reasoning



## **Program-Aided LMs (PAL)**

- Due to **autoregression**, LLMs are not adept at numerical calculations.
- Transforming reasoning into NL2Code.
- Compute via, not "predicting the next token."

A "Two-stage" approach: understanding + coding

#### Chain-of-Thought (Wei et al., 2022)

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

#### A: Roger started with 5 tennis balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

Q: The bakers at the Beverly Hills Bakery baked 200 loaves of bread on Monday morning. They sold 93 loaves in the morning and 39 loaves in the afternoon. A grocery store returned 6 unsold loaves. How many loaves of bread did they have left?

Model Output

Input

A: The bakers started with 200 loaves. They sold 93 in the morning and 39 in the afternoon. So they sold 93 + 39 = 132 loaves. The grocery store returned 6 loaves. So they had 200 - 132 - 6 = 62 loaves left. The answer is 62.

#### Program-aided Language models (this work)

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: Roger started with 5 tennis balls. tennis\_balls = 5 2 cans of 3 tennis balls each is bought\_balls = 2 \* 3 tennis balls. The answer is answer = tennis\_balls + bought\_balls

Input

Q: The bakers at the Beverly Hills Bakery baked 200 loaves of bread on Monday morning. They sold 93 loaves in the morning and 39 loaves in the afternoon. A grocery store returned 6 unsold loaves. How many loaves of bread did they have left?

Model Output A: The bakers started with 200 loaves loaves\_baked = 200 They sold 93 in the morning and 39 in the afternoon loaves\_sold\_morning = 93 loaves\_sold\_afternoon = 39 The grocery store returned 6 loaves. loaves\_returned = 6 The answer is answer = loaves\_baked - loaves\_sold\_morning - loaves\_sold\_afternoon + loaves\_returned >> print(answer)

# Background



## **Common Issues in Reasoning with CoT**

Question: There are 700 bees in a hive. There are twice as many worker bees as baby bees, and there are twice as many babies as queens. How many worker bees are there? Correct Answer: 400

- Due to model limitations, it might not necessarily understand the problem.

- Using CoT can lead to **cumulative errors**.



Misunderstanding the relationship between variables.

# Background



## **Common Issues in Reasoning with CoT**

Question: There are 700 bees in a hive. There are twice as many worker bees as baby bees, and there are twice as many babies as queens. How many worker bees are there? Correct Answer: 400

- Despite using CoT, it doesn't fundamentally solve LLM's computational issues.
- Especially when dealing with large number calculations.



Wrong calculation.





### **Common Issues in Reasoning**

Question: There are 700 bees in a hive. There are twice as many worker bees as baby bees, and there are twice as many babies as queens. How many worker bees are there? Correct Answer: 400

- For PAL, the model must both correctly understand the question and write the correct code.

- Excel at large number calculations, but it adds some complexity to reasoning.



Codes fail to accurately reflect the problem statement.

# **Existing Problems**



## Perspective from a single LLM

## 1. Hallucinations Problems

- 2. Context length [3]
- 3. Self-Refine/Correction might not always work [4,5]

Solo-Pe	rformance-Prompting / prompts / codenames_collaborative.py	Symbols ×		
Code	Blame 526 lines (379 loc) · 34.4 KB		Ø	Find definitions and references for functions and other symbols in this file by clicking a symbol below or in the
19	spp_prompt_spymaster = '''When faced with a task, begin by identifying the participants who will contribute to solving	the task.	Then,	code. 🕢 Summarize : 🗙
41	ranam coccuprocess.			= Filter symbols
42	Final answer: 6 * (1 + 1) + 12 = 24			
43				const standard_prompt_spyma
44				const cot prompt spymaster
45	Example Task 2: Write a poem that meets the following requirements: (1) the poem has seven lines and the first letters	of each l	ine fo	cot_prompt_opymaster
46				const spp_prompt_spymaster
47	Participants: AI Assistant (you); Poet; Computer Scientist; Ten year old child			const spp prompt spymaster f
48				
49	Poet: Make sure that you write the poem with seven lines, and the first letters of the lines should be C, H, A, T, G, F	Р, Т.		const spp_prompt_spymaster
50	computer scientist: A quantum computer is an advanced computing device that uses the principles of quantum mechanics to	o process	and sto	const standard_prompt_guesser
52	ten year old childs i hope for the puer to be for and easy to dimensionaling. I don't want to see a for of jargons of completent and easy to dimensionaling in the norm:	reaced con	cepts.	
53	a assistant your, many or che galance, here's my inicial accempt at the polm. Computational worder of our age.			const cot_prompt_guesser
54	Harnessing the quantum world's strange ways.			const spp_prompt_guesser
55	Atoms dance, entwined in dual state,			
56	Tapping secrets hidden in their haze.			const spp_prompt_guesser_fixe
57				const spp_prompt_guesser_pro
58	Grand power to solve, simulate,			
59	Profound problems that perplex the wise,			
60	Transforming our future, we await.			
61				
62	Poet: Let's verify if the poem meets the requirements. The first letters are CHATGPT which is correct! And the poem rhy	ymes well.	Good	
64	computer sciencist: everything tooks good to me:			
65	The second click. I don't know what does perpick mean, can you make the use of words easier to unerstand.			
66	Curious machine of our time.			
67	Harnessing the quantum realm's odd ways,			
68	Atoms play, two states they embrace,			
69	Taking secrets from their puzzling maze.			
70				
71	Great power to solve and imitate,			
72	Problems that confuse the brightest minds,			
73	Fransforming our future, we await.			
74	Reat: Latte chark again if the near meets the requirements. The first latters are C H A T G B T. And new the near is m		ible to	
76	Formular Scientist: Looking agod	ore access	The c	
77	Ten year old child: I like this version a lot!			
78				
79	Finish collaboration!			
80				
81	Final answer:			
82	Curious machine of our time,			6
83	Harnessing the quantum realm's odd ways,			e
84	Atoms play, two states they embrace,			
85	Taking secrets from their puzzling maze.			
86	Creat areas to calve and initiate			
8/	ureat power to solve and imitate,			
00 89	Transforming our future, we await.			
09	fransforming our facare, we aware			

#### Unacceptable Prompt Length [3]

[3] Unleashing the Emergent Cognitive Synergy in Large Language Models: A Task-Solving Agent through Multi-Persona Self-Collaboration, 2023

[4] GPT-4 Doesn't Know It's Wrong: An Analysis of Iterative Prompting for Reasoning Problems, 2023

[5] Can Large Language Models Really Improve by Self-critiquing Their Own Plans?, 2023

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# **Existing Problems**

## **Perspective from methodologies**

# **Self-Consistency Decoding**

- Based on majority voting mechanism [6].
- Correct ones overwhelmed by incorrect ones
- At its core, it's still a single model operating on its own.
- Very expensive

# **Other Prompting Strategies**

- Complexity-based Prompting [7] / PHP [8] / ...
- Curated & task-specific prompts required!
- Also not cost-effective



There are 700 bees in a hive. There are twice as many Ouestion worker bees as baby bees, and there are twice as many babies as queens. How many worker bees are there? If there are twice as many babies as queens, then the number of queens is  $700 / (2 \ge 2) = 175$ . Then the number of worker bees is  $2 \times (700 - 175) = 1050$ . Given that Q represents the number of queens in the hive, then Q + 2Q + 4Q = 700. Solving the equation yields Q = 100. Consequently, the number of worker LLMs bees can be calculated as 4Q, resulting in 4 \* 100 =400 worker bees.  $\mathbf{P}$ If the number of babies is twice that of queens, then the number of queens can be calculated as  $700 / (2 \times 2)$ = 175. The number of worker bees can be determined as  $2 \ge (700 - 175) = 1050$ .

[6] Self-Consistency Improves Chain of Thought Reasoning in Language Models, 2023

[7] Complexity-Based Prompting for Multi-Step Reasoning, 2023

[8] Progressive-Hint Prompting Improves Reasoning in Large Language 8 Models, 2023

# **Motivations**



## Two heads are better than one!

## Collaborate! Let models take on different roles to solve problems together.



Discuss

Review

Retrieve



**Discuss:** Enhances factuality, avoids common answer bias, and preserves diverse thoughts among agents.

- LLMs are split into **two Teams** and **one judge**.
- Teams undergo **iterative discussions** to refine reasoning & predictions.
- Consistent agreement ends the debate; Any discrepancy leads to judge intervention.
- The judge evaluates the quality of reasoning and predictions to make an informed decision.

\* Different from previous "Debates" [9,10]

Question: Vincent bought 10 books about animals, 1 book about outer space, and 3 books about trains. Each book cost \$16. How much did Vincent spend on the books?



[9] Encouraging divergent thinking in large language models through multiagent debate, 2023

[10] Improving factuality and reasoning in language models through multiagent debate, 2023



# Discuss: Comparison and a case study (one round)

- Identify errors that single model misses
- Correct them on the spot!
- Stronger than previous methods

Method	MultiArith	SingleEQ	AddSub
MAD	96.9	93.8	88.6
EoT	99.0	<u>94.3</u>	88.0
Corex-Discuss	98.7	93.7	<u>89.7</u>
Corex-Retrieve	<b>99.8</b>	<u>94.3</u>	<b>90.9</b>
Corex-Review	<u>99.5</u>	<b>96.9</b>	89.6

**Q**: Mark took a test yesterday that consisted of 75 questions. He completed the test at a rate of 5 questions per hour. Today, he took another test of 100 questions at the same rate. If Mark had 8 hours to complete the first test and 6 hours to complete the second one, how many questions did he leave incomplete?

. . .

# Mark completed the first test at a rate of 5 questions per hour, and he had 8 hours to complete it. So he completed 8 \* 5 = 40 questions. He completed the second test at the same rate, 5 questions per hour, and he had 6 hours to complete it. So he completed 6 \* 5 = 30 questions. Therefore, Mark completed a total of 40 + 30 = 70 questions. Since the first test had 75 questions and Mark completed 70 questions, he left 75 - 70 = 5 questions incomplete. Therefore, Mark left 5 questions incomplete.



Jerry's analysis is mostly correct, but there is a small mistake in the final cal-

culation. Let's break down the solution step by step: From the given information: Mark completed the first test at a rate of 5 questions per hour, and he had 8 hours to complete it. So he completed 8 \* 5 = 40 questions on the first test. He completed the second test at the same rate, 5 questions per hour, and he had 6 hours to complete it. So he completed 6 \* 5 = 30 questions on the second test. Therefore, Mark completed a total of 40 + 30 = 70 questions. However, the mistake was made in the next step: Since the first test had 75 questions and Mark completed 40 questions, the number of questions he left incomplete on the first test is 75 - 40 = 35 questions. Similarly, the second test had 100 questions and Mark completed 30 questions, so he left 100 - 30 =70 questions incomplete on the second test. To find the total number of questions left incomplete, we need to add the number of questions left incomplete on both tests: 35 + 70 = 105. So, the answer is Mark left 105 questions incomplete.

. . .



# **Review:** Mitigating common issues faced by CoT / PAL

## - Recap the Drawbacks:

- CoT: Cumulative errors and quality plateau.
- PAL: Misinterpretation and potential for buggy codes.

## - Introducing the Review mode for collaborative error-checking:

- One LLM initiates reasoning chains, predictions, and code.
- Other agents sequentially review and refine the solution.

## - Review process:

- Each iteration improves upon the feedback of the previous one.
- Final iteration determines the outcome; code executed by Python interpreter.



Agents 1 and 3 are correct.



## **Review:** A case study

Input

Q: There are 15 trees in the grove... A: There are 15 trees originally...

#### • • •

Q: There are 700 bees in a hive. There are twice as many worker bees as baby bees, and there are twice as many babies as queens. How many worker bees are there?

#### Output

Let's start with the number of queens. If there are twice as many babies as queens, then there are 700 / 4 = 175queens. If there are twice as many worker bees as baby bees, then there are 2 \* (700 / 4) = 350 worker bees. The answer is  $350.\times$ 

(a) Chain-of-Thought

Input

Q: There are 15 trees in the grove... A: There are 15 trees originally...

•••

Q: There are 700 bees in a hive. There are twice as many worker bees as baby bees, and there are twice as many babies as queens. How many worker bees are there?

Output

If there are twice as many babies as queens, then the number of queens is  $700 / (2 \ge 2) = 175$ . Then the number of worker bees is  $2 \ge (700 - 175) = 1050$ .

Given that Q represents the number of queens in the hive, then Q + 2Q + 4Q = 700. Solving the equation yields Q = 100. Consequently, the number of worker bees can be calculated as 4Q, resulting in 4 \* 100 = 400 worker bees.

If the number of babies is twice that of queens, then the number of queens can be calculated as  $700 / (2 \ge 2) = 175$ . The number of worker bees can be determined as  $2 \ge (700 - 175) = 1050$ .

#### (b) SC Decoding

Input

You are currently reviewing a math problem solved by Alice...

Q: There are 700 bees in a hive. There are twice as many worker bees as baby bees, and there are twice as many babies as queens. How many worker bees are there? Alice: I think ...

Output

Alice's solution is incorrect. She divided the total number of bees (700) by 4, assuming that the queens make up one quarter of the total. By denoting the number of queen bees as x, we can express the number of baby bees as 2x and the number of worker bees as 4x. By solving for x, we get x=100, which means there are 400 worker bees.

(c) Corex-Reviewers



**Retrieve:** Focuses on the alignment between reasoning chains and answers, rather than just predictions.

## **Recap some drawbacks:**

- Risk of correct answers being influenced by incorrect ones.
- Performance saturation with computational burden.

## **Retrieve process:**

- An LLM acts as the retriever. Remaining ones perform reasoning on a query, forming a candidate pool *P*.

- Retriever evaluates the faithfulness **between chains and predictions** from *P*, assigning a confidence score.

- Response(s) with the highest confidence will be the final answer.

0	Question: Which describes the composition of carbohydrates? (A) lipids bonding to form phospholipids (B) monomers bonding to form polymers (C) amino acids bonding to form polypeptides (D) saccharides bonding to form polysaccharides
	<b>Agent1:</b> The composition of carbohydrates involves saccharides bonding to form polysaccharides. So the answer is (D).
8	Agent2: The composition of carbohydrates is described by option (C), amino bonding to form polysaccharides. So the answer is (C).
-	<b>Agent3:</b> The correct answer is (D) saccharides bonding to form polysaccharides. Carbohydrates are composed of monosaccharides that can join together through a process called dehydration synthesis to form larger polysaccharides. So the answer is (D).
ĕ	Agent4: Option (C), saccharides bonding to form polypeptides, describes the composition of carbohydrates. So the answer is (C).
9	Agent5(Retriever): The confidence scores are: [0.9, 0.2, 0.9, 0.1]. Agents 2, 4 made errors in their descriptions. Specifically, Agent 2 confused amino acids with saccharides, while Agent 4 incorrectly stated that saccharides bond to form polypeptides. In reality, saccharides bond to form polysaccharides.

Thus, they are not faithful in their responses.

Agents 1 and 3 are correct.

# **Experiments**



# Mathematical Reasoning / Semi-structured Reasoning

	GSM8K	SVAMP	MultiArith	SingleOP	SingleEQ	AddSub	GSM-Hard	Avg.		FinQA	ConvFinQA	Avg.
CoT	74.5	78.9	98.5	94.1	93.3	87.8	39.0	80.9	СоТ	46.1	50.4	48.3
ComplexCoT	79.7	80.7	97.3	94.3	92.3	86.8	39.7	81.5	CoT-SC(10)	52.7	57.2	54.9
CoT-SC(10)	82.8	84.5	99.8	<u>95.4</u>	95.1	89.6	45.2	84.6	PAL	54.3	50.8	52.9
PAL	76.0	83.4	96.7	90.7	95.8	87.6	<u>62.1</u>	84.6		54.5	50.0	52.7
Corex-Debate	76.2	82.6	98.7	94.8	93.7	89.7	45.9	83.1	Corex-Debate	50.2	56.7	53.5
Corex-Review <sub>NT</sub>	80.3	83.2	99.5	95.0	94.3	$\frac{89.4}{89.4}$	50.8	84.6	Corex-Review <sub>NL</sub>	52.5	52.3	52.4
Corex-Review <sub>Code</sub>	79.2	85.8	98.3	93.6	96.9	89.6	63.6	86.7	Corex-Review <sub>Code</sub>	55.9	54.2	55.1
Corex-Retrieve	82.5	85.6	99.8	96.1	96.6	90.9	53.0	86.3	Corex-Retrieve	<u>55.4</u>	57.7	56.6

## Experiments under GPT-3.5 Turbo

- Surpassing CoT-SC(10) while only using five LLMs.

- Considering both two scenarios for vary problem complexity.

# **Experiments**



# More reasoning scenarios

		Date	Penguin	Colored Objects	Repeat Copy	Avg.
	СоТ	82.0	81.5	88.0	43.8	73.8
Also works wall on Commonsonso / Symbolic tasks	CoT-SC(10)	87.9	86.2	94.8	53.1	80.5
- Also works well on Commonsense / Symbolic tasks	PAL	81.2	91.3	86.8	<u>93.8</u>	88.3
	Corex-Debate	83.2	85.9	91.2	62.5	80.7
- Has the potential to be a generic reasoning solution	Corex-Review <sub>NL</sub>	84.0	92.0	92.4	59.4	82.0
This the potential to be a generic reasoning solution.	Corex-Review <sub>Code</sub>	$Xeview_{NL}$ 64.092.092.459.4 $Review_{Code}$ 82.793.391.696.9	96.9	91.1		
	Corex-Retrieve	<u>84.6</u>	92.6	95.6	68.8	85.6

	StrategyQA	CSQA	OpenBookQA	BoolQ	ARC-c	Avg.
СоТ	65.3	76.7	82.6	65.1	84.2	74.8
ComplexCoT	63.1	77.5	-	-	-	-
CoT-SC(10)	67.1	<u>78.1</u>	<u>85.2</u>	66.6	85.7	76.5
Corex-Debate	68.4	78.9	83.4	66.9	86.3	76.8
Corex-Review <sub>NL</sub>	66.9	77.4	84.8	66.9	86.0	76.4
Corex-Retrieve	69.3	77.7	87.6	<b>68.0</b>	85.5	77.6

# Analysis



# **Delving into LLMs' behaviors**

# **1. LLMs as Judges in Debate Mode:**

Performance is influenced by the judge's LLM type.
As task complexity rises, the capability of the judge becomes more crucial.

## 2. LLMs as Retrievers in Retrieve Mode:

- Model choice has a subtle impact on performance.
- Performance is capped by the capabilities of the candidates.
- Even with less powerful models as retrievers, satisfactory results can be attained.



Figure 8: Comparison of using different LLMs as judges in Debate mode.



Figure 9: Comparison of using different LLMs as retrievers in Retrieve mode.

# Analysis



# **Reaching Cost-Effectiveness**

1. *Corex* achieve similar or better results with reduced costs.

2. Requires **only 5-10%** of the computational resources used by majority voting methods.

#### 91 Retrieve ComplexCoT(40) Review Accuracy(%) 88 68 66 06 CoT-SC(10)CoT-SC(20) CoT-SC(40)<sup>CoT-SC(80)</sup> Debate 👝 CoT-SC(5) PAL ComplexCoT(20) ComplexCoT(10) CoT ComplexCoT 87 6.25 6.50 4.25 4.50 4.75 5.00 5.25 5.50 5.75 6.00 Tokens Consumption in $log_{10}$ Scale

AddSub Dataset

\* Also have more features like annotation-efficiency ...

# Analysis



# Synergies among different strategies

- *Corex* methods can be integrated easily, even combining with other approaches like self-refine.

- Performance can be further enhanced after combining.





- 1. Automated search for collaboration methods.
- 2. Combine external feedback for collaboration
- 3. Extend multi-agent task solving to VLM scenarios [11].





# Thanks